Effects of measurements on quantum evolution and the time-energy uncertainty relation

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The quantum Zeno effect (QZE) is the striking possibility of inhibiting the decay of an unstable quantum state by sufficiently frequent measurements. This effect is currently believed to be universal, even though it has only been observed for oscillations between discrete states. Recently we showed that both the QZE and the inverse (anti-) Zeno effect (AZE) can be observed on spontaneous decay in cavities [1].

Here we develop a general theory of evolution of quantum systems in the presence of frequent measurements [2]. We consider both the ideal case of instantaneous projections of the wave function to the initial state and specific schemes of impulsive and continuous measurements. In all the cases considered the initial state decays with the rate, which can be cast in the following universal form,

$$R = 2\pi \int G(\omega)F(\omega)d\omega. \tag{1}$$

Here the function $G(\omega)$ is the spectral density of the strength of the coupling between the initial state and other states $|j\rangle$. Equation (1) holds for any number of states $|j\rangle$. It applies both to the case of one level $|j\rangle$, as in the notorious experiment of Ref. [3], and to the case when the states $|j\rangle$ belong to a spectrally dense band ("reservoir"). The form factor $F(\omega)$ (normalized to one) in (1) accounts for the measurement-induced broadening and shift of the initial level. It appears that, while the shape of $F(\omega)$ depends on a specific scheme, the width ν of $F(\omega)$ provides generally the effective measurement rate. In particular, $\nu \sim 1/\tau$ for ideal measurements performed with equal intervals τ. Equation (1), thus, implies the energy-time uncertainty relation $\Delta E \Delta t \sim \hbar$, where ΔE is the width of level $|e\rangle$ and $\Delta t \sim 1/\nu$. The above considerations imply the impossibility of the continuous-observation limit $\nu \to \infty$, necessary to stop quantum evolution (the quantum Zeno paradox). Indeed, this limit leads to an unbounded increase of the energy fluctuations of the system, which would cause the system to disintegrate.

The present theory describes different measurement effects, including the QZE and AZE, and allows us to obtain the general criteria of their validity. In particular, we show that the AZE, rather than the QZE, is achievable for spontaneous decay of atoms in free space.

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[3] W. M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland, *Phys. Rev. A* **41**, 2295 (1990).